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ON A CERTAIN QUARTIC CURVE WHICH MAY DEGENERATE INTO AN ELLIPSE.*

By R. D. CARMICHAEL.

The object of this paper is the discussion of a class of quartic curves whose equation may be put in the form

$$\nu\sqrt{[x^2+y^2]}\pm\mu\sqrt{[x^2+(a-y)^2]}=\lambda. \tag{1}$$

When the equation is cleared of radicals the double sign ± can be made to disappear entirely. In the following discussion it will be shown that this curve consists of two ovals. As a limiting case, when $\mu=\nu$ one of the ovals vanishes and the other becomes an ellipse.

§1. CONSTRUCTION OF THE LOCUS BY CONTINUOUS MOTION.

Take two points A and B at a distance a apart, and let A be the origin of rectangular axes, and let the y-axis pass through B. Take the point P such that

$$\mu.PB+\nu.PA=\lambda. \tag{2}$$

Then P is a point on the locus; for evidently it satisfies the equation when the double sign is taken *plus.* Hence the branch on which P is located may be defined as the locus of a point which moves so that the sum of μ times its distance from one point and ν times its distance from another is constant. We may therefore devise the following method for constructing this branch when μ and ν are commensurable.

Divide equation (1) by such common factor of μ and ν as will make the respective quotients m and n relatively prime integers, and let the quotient of λ divided by this factor be l. Then the equation becomes

$$n\sqrt{[x^2+y^2]}\pm m\sqrt{[x^2+(a-y)^2]}=l. \tag{3}$$

For the point P we now have

$$m.PB+n.PA=l. \tag{4}$$

One or both of the numbers m, n are odd. Consider m odd and n even.

Attach a string at B and pass it around a pencil point at P, wrapping until m plies of the cord pass from B to P. Then pass the free end around A, wrapping until n plies pass from A to P. Then attach the cord to the

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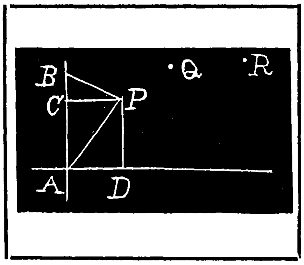
[*] Presented to the Chicago Section of the American Mathematical Society, March 30, 1907.



pencil at P so that every part of it is drawn tight. Now if the pencil so moves that the chord is kept tightly stretched while it slides around A, B, and P, P will describe one branch of the curve. (The modifications necessary for n odd or m even are evident). We shall next give a method for constructing the other branch by continuous motion. For this case we shall employ

$$(5) \qquad n\sqrt{[x^2+y^2]}-m\sqrt{[x^2+(a-y)^2]}=l.$$

If Q in the figure is so taken that

$$(6) \qquad n.AQ-m.BQ=l,$$

Q is evidently a point on this second branch.

Now, pass a cord around the point at B and the pencil at Q so that m plies pass from B to Q. If m is odd one end of the string is to be attached at B; while if m is even it is to be fastened to the pencil point at Q. The free end is to pass out in some direction as QR. In like manner join Q and A with another chord having its free end in the direction QR. Of this cord n plies are to pass from A to Q. Let these two chords be tightly stretched and tied together at some convenient point. If Q moves so as to keep these conditions always fulfilled, it is evident that the pencil will mark on the paper the second branch of this curve.

From the manner of construction it is evident that each of these branches is closed. Such a branch we shall as usual call an oval. In the next section we shall show that these two are the only branches of the locus.

§2. GENERAL NATURE OF THE LOCUS.

As the two branches of the locus have the respective equations

$$\nu\sqrt{[x^2+y^2]}+\mu\sqrt{[x^2+(a-y)^2]}=\lambda,$$
$$\nu\sqrt{[x^2+y^2]}-\mu\sqrt{[x^2+(a-y)^2]}=\lambda,$$

it is evident they have no point in common except when

$$\mu\sqrt{[x^2+(a-y)^2]}=0.$$

Now μ is not zero; for then the locus would reduce to a circle. Hence

$$x^2+(a-y)^2=0.$$

Since each term of the first member is positive the equation can be satisfied only for $x^2=0$ and $(a-y)^2=0$. Hence, $x=0$, $y=a$. That is, the two branches of the curve can have no point in common except $(0, a)$. Moreover, this point is on the locus only when $\nu a=\lambda$, as may be seen by substituting in equation (1). When this condition is satisfied the two branches are tangent at the point $(0, a)$; for if they should cut each other here they would necessarily have another point in common. Moreover one branch is always entirely within the other.

Now if there is another branch to the locus besides the two already discussed some line passing through the inner of these two ovals cuts this third branch. The line then intersects the *quartic* curve in *five* points; but this is impossible. Hence the entire locus consists of the two ovals above constructed.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

276. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

If $x_1, x_2, x_3, \ldots, x_n$ be unequal, and $f(x)$ be a rational integral function of degree $\not> n-2$, then shall

$$\sum_{r=1}^{r=n-1} \frac{f(x_r)}{(x_r-x_1)(x_r-x_2)\ \ldots\ (x_r-x_n)}=0.$$

No solution of this problem has been received. It is not clear what is intended in the problem. By taking $f(x)=x^2-x+1$ and $x_1=1, x_2=2, x_3=3, x_4=4$, the problem is not true according to our interpretation of it, since infinite addends would be introduced.

In Burnside and Panton's *Theory of Equations*, p. 319, 3rd edition, example 4, slightly changed, reads: If the degree of $\phi(x)$ does not exceed $n-2$, prove that $\sum_{r=1}^{r=n} \frac{\phi(x_r)}{f'(x_r)}=0, \ldots, x_r (r=1, 2, \ldots, n)$ being roots of $f(x)=0$.

Additional information on this problem is desired. ED. F.

277. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

If $\alpha, \beta, \gamma, \delta$ are the roots of the quartic $ax^4+bx^3+cx^2+dx+e=0$, calculate the value of the product of the twelve expressions of the form $(4\alpha-2\beta-\gamma-\delta)$ in terms of H, I, J, the well known functions of the differences of the roots.